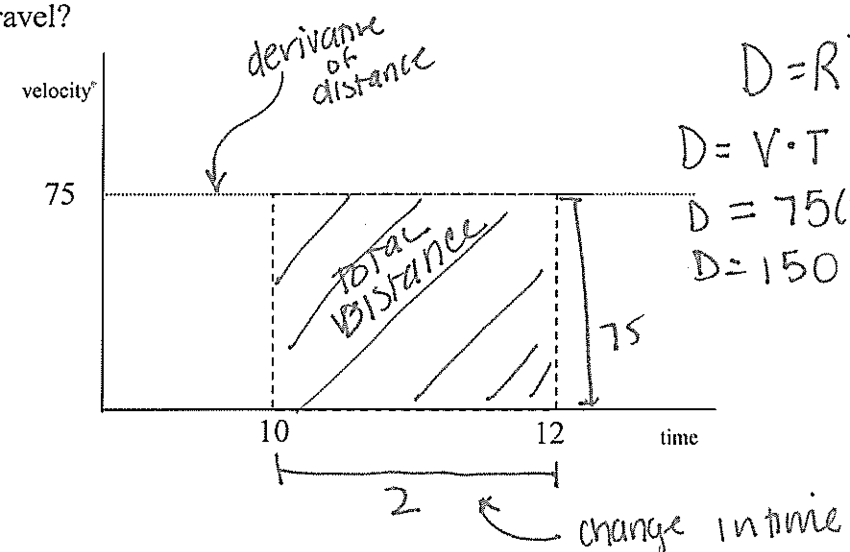
Name ______Key______ AP Calculus AB
Chapter 5 Day 1

Estimating with Finite Sums (5.1)

Up to this point we've dealt with differential calculus (involving the concept of tangents to curves and derivatives). We will now move on to integral calculus, which deals primarily with areas under curves.

Example: If I travel at 75 mph (constant velocity) between 10 am and noon how far did I travel?



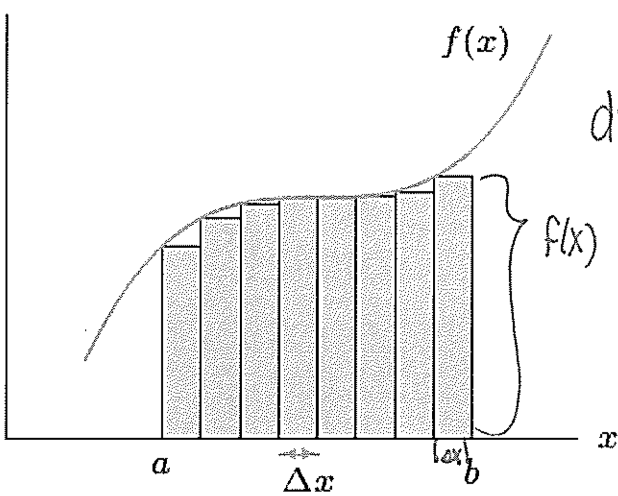
$D = RT$

$D = V \cdot T$

$D = 75(2)$

$D = 150$ miles

But what if the velocity was not constant?



draw more rectangles!

$f(x) \cdot \Delta x$

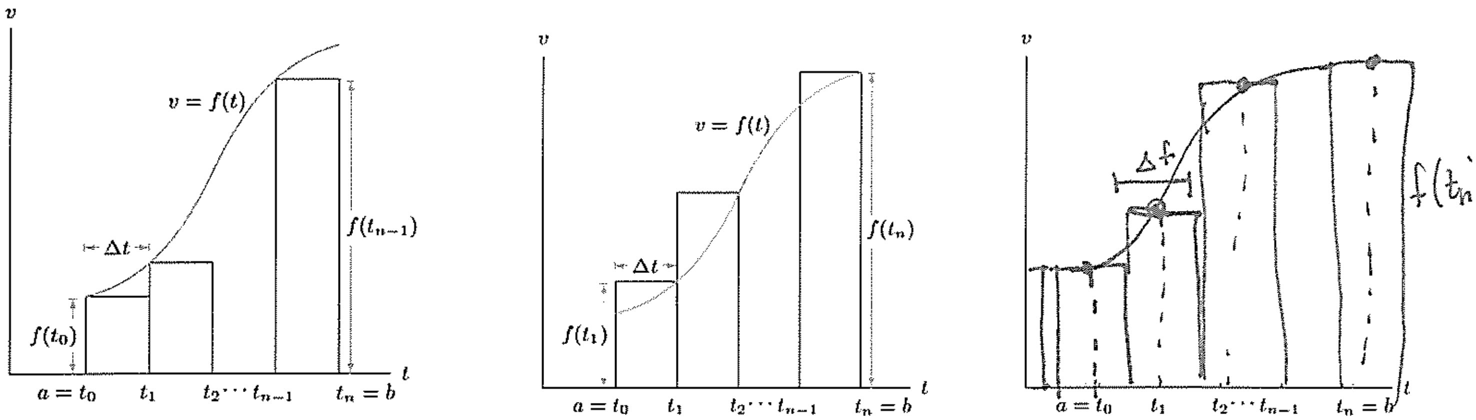
Rectangular Approximation:

Steps: 1) Divide the interval in question into n subintervals

2) The width of each rectangle is $\frac{\text{length of interval}}{n}$

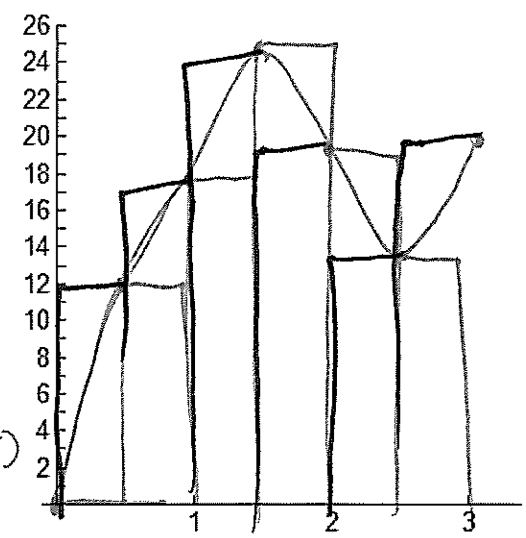
3) Find the midpoint, left endpoint or right endpoint of each region, depending on which specific method you want to use.

4) The height of the rectangle in each subinterval will be the value of the function at the midpoint, left endpoint or right endpoint.



Example 1 Given a table: A bicyclist monitored her speed over the course of three hours. Estimate the total distance traveled.

t	v
0	0
.5	12
1	18
1.5	25
2	20
2.5	14
3	20



(A) LEFT(6)

$0(.5)+12(.5)+(18)(.5)+(25)(.5)+20(.5)$
$+14(.5)=44.5$

(B) RIGHT (6)

$12(.5)+18(.5)+25(.5)+20(.5)+14(.5)+20(.5)=54.5$

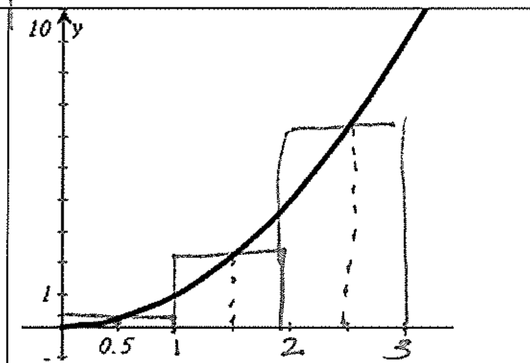
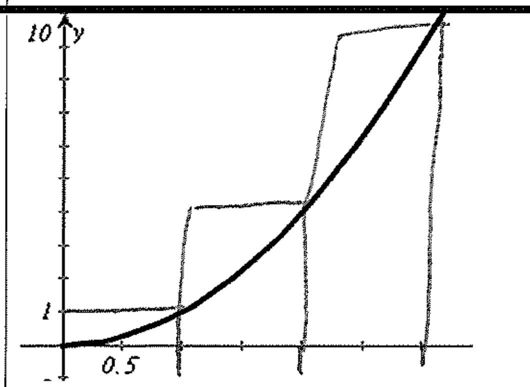
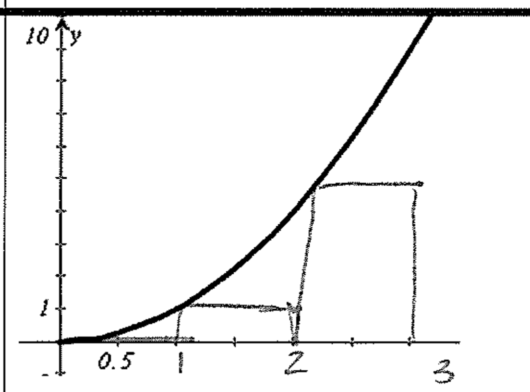
(C) MID(3)

$(1)(12)+(1)(25)+(1)(14)=51$

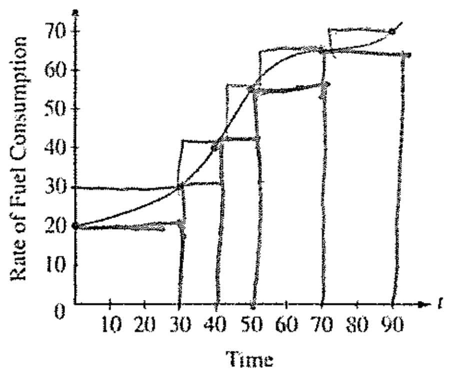
Example 2 Given interval length and number of subintervals:

A particle starts at $x = 0$ and moves along the x-axis with velocity $v(t) = t^2$ for $t \geq 0$.
How far did the particle travel when $t = 3$, if we use n=3 subintervals? $\frac{0-3}{3} = 1$

t	0	.5	1	1.5	2	2.5	3
$v(t)$	0	$\frac{1}{4}$	1	$\frac{9}{4}$	4	$\frac{25}{4}$	9

MRAM n = 3 Midpoint Rectangular Approximation Method MID (3)		Area of Rectangle 1 = $(1)(.5)^2 = .25$ Area of Rectangle 2 = $(1)(1.5)^2 = 2.25$ Area of Rectangle 3 = $(1)(2.5)^2 = 6.25$ TOTAL AREA =
RRAM n =3 Right endpoint Rectangular Approximation Method RIGHT (3)		Area of Rectangle 1 = $(1)(1)^2 = 1$ Area of Rectangle 2 = $(1)(2)^2 = 4$ Area of Rectangle 3 = $(1)(3)^2 = 9$ TOTAL AREA = 14
LRAM n = 3 Left-endpoint Rectangular Approximation Method LEFT(3)		Area of Rectangle 1 = $(1)(0) = 0$ Area of Rectangle 2 = $(1)(1)^2 = 1$ Area of Rectangle 3 = $(1)(2)^2 = 4$ TOTAL AREA = 5

Example 3: Given a table, unequal intervals.



t (minutes)	$R(t)$ (gallons per minute)
0	20
30	30
40	40
50	55
70	65
90	70

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice differentiable and strictly increasing function R of time t. The graph of R and a table of selected values of R(t), for the time interval $0 \le t \le$ 90 minutes, are shown above.

(a) Approximate the value of the total fuel consumed in the first 90 minues using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the actual value? Explain your reasoning.

$20(20)+(10)(30)+10(40)+20(55)+(20)(65)=3700$ gallons

Less than since the rectangles are under the curve.

(b) Approximate the value of the total fuel consumed in the first 90 minues using a right Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the actual value? Explain your reasoning.

$(20)(30)+(10)(40)+20(55)+20(70)+20(65)=4550$g

more than since the rectangles are above the curve

Example 4:

A student is speeding down Route 1 in his fancy red Porsche when his radar system warns him of an obstacle 400 feet ahead. He immediately applies the breaks, starts to slow down and spots a skunk in the road directly ahead of him. The "black box" in the Porsche records the car's speed every two seconds producing the following table.

Time since brakes applied (sec)	0	2	4	6	8	10
Speed (ft/sec)	100	80	50	25	10	0

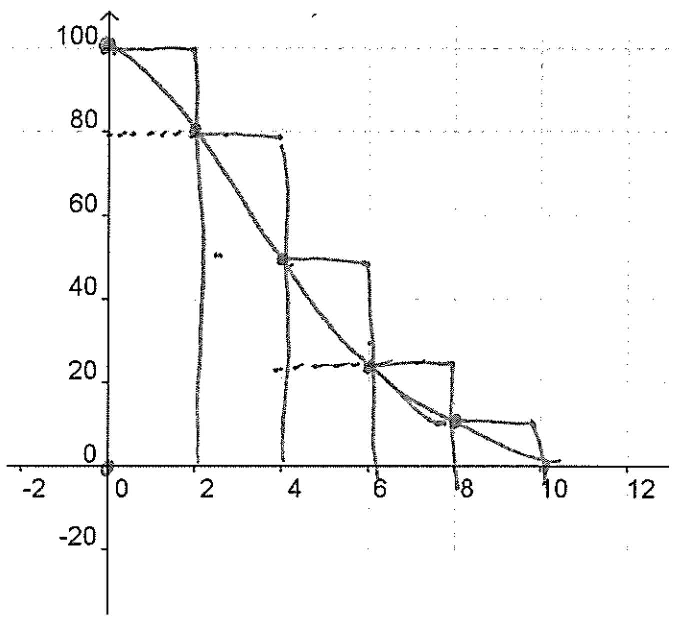
(a) What is do you think would be the best estimate of the distance traveled?

$LH = (100)(2) + 80(2) + (50)(2) + 25(2) + 10(2) = 530 ft$

$RH = 80(2) + 50(2) + 25(2) + 10(2) = 330$

$MID = 80(4) + 25(4) = 420$

$RH \leq Actual \leq LH$



(b) Which one of the following statements can you justify from the information given?

(i) The car stopped before getting to the skunk.

(ii) The black box data is inconclusive. The skunk may or may not have been hit.

(iii) The skunk was hit by the car.

Homework: p254 1-3, 5-10, 13