3.4b

2. $f(x) = \sqrt{x}\sin(x) = x^{\frac{1}{2}}\sin(x)$

$$f'(x) = x^{\frac{1}{2}}\cos(x) + \sin(x)\cdot\left(\tfrac{1}{2}x^{-\frac{1}{2}}\right)$$
$$= x^{\frac{1}{2}}\cos(x) + \tfrac{1}{2}x^{-\frac{1}{2}}\sin(x)$$

4. $y = 2\csc(x) + 5\cos(x)$

$$y' = 2[-\csc(x)\cot(x)] + 5[-\sin(x)]$$
$$= -2\csc(x)\cot(x) - 5\sin(x)$$

6. $g(t) = 4\sec(t) + \tan(t)$

$$g'(t) = 4[\sec(t)\tan(t)] + \sec^2(t)$$
$$= 4\sec(t)\tan(t) + \sec^2(t)$$

8. $y = u(a\cos(u) + b\cot(u))$ NOTE: u is variable

Product Rule

$$y' = u[a(-\sin(u)) + b(-\csc^2(u))] + [a\cos(u) + b\cot(u)]\cdot 1$$
$$= -au\sin(u) - bu\csc^2(u) + a\cos(u) + b\cot(u)$$

10. $y = \dfrac{1+\sin(x)}{x+\cos(x)}$

$$y' = \frac{[x+\cos(x)]\cdot\cos(x) - [1+\sin(x)][1-\sin(x)]}{[x+\cos(x)]^2}$$

$$= \frac{x\cos(x) + \cos^2(x) - [1-\sin^2(x)]}{[x+\cos(x)]^2}$$

$$= \frac{x\cos(x) + \cos^2(x) - 1 + \sin^2(x)}{[x+\cos(x)]^2}$$

Note: $\sin^2(x) + \cos^2(x) = 1$

$$= \frac{x\cos(x) - 1 + 1}{[x+\cos(x)]^2}$$

12. $y = \dfrac{1-\sec(x)}{\tan(x)}$

$$y' = \frac{\tan(x)[0-\sec(x)\tan(x)] - [1-\sec(x)]\sec^2(x)}{\tan^2(x)}$$

$$= \frac{-\sec(x)\tan^2(x) - \sec^2(x) + \sec^3(x)}{\tan^2(x)}$$

$$= \frac{\sec(x)[-\tan^2(x) - \sec(x) + \sec^2(x)]}{\tan^2(x)}$$

Note: $\tan^2(x) + 1 = \sec^2(x)$

$\therefore \sec^2(x) - \tan^2(x) = 1$

$$= \frac{\sec(x)[1-\sec(x)]}{\tan^2(x)}$$

14. $y = \csc(\theta)[\theta + \cot(\theta)]$

Product Rule

$$y' = \csc(\theta)[1 - \csc^2(\theta)] + [\theta + \cot(\theta)][-\csc(\theta)\cot(\theta)]$$
$$= \csc(\theta) - \csc^3(\theta) - \theta\csc(\theta)\cot(\theta) - \csc(\theta)\cot^2(\theta)$$
$$= \csc(\theta)[\underline{1 - \csc^2(\theta)} - \theta\cot(\theta) - \cot^2(\theta)]$$

Note: $1 + \cot^2(\theta) = \csc^2(\theta)$

$\therefore 1 - \csc^2(\theta) = -\cot^2(\theta)$

$$= \csc(\theta)[-\cot^2(\theta) - \theta\cot(\theta) - \cot^2(\theta)]$$
$$= \csc(\theta)[-2\cot^2(\theta) - \theta\cot(\theta)]$$
$$= \cot(\theta)\csc(\theta)[-2\cot(\theta) - \theta]$$

16. $y = x^2\sin(x)\tan(x) = [x^2\sin(x)][\tan(x)]$

Product Rule

$$y' = [x^2\sin(x)]\frac{d}{dx}\tan(x) + \tan(x) \cdot \underbrace{\frac{d}{dx}[x^2\sin(x)]}_{\text{Product Rule}}$$

$$y' = x^2\sin(x) \cdot \sec^2(x) + \tan(x)[x^2 \cdot \cos(x) + \sin(x) \cdot 2x]$$
$$= x^2\sin(x)\sec^2(x) + x^2\underbrace{\cos(x)\tan(x)}_{\frac{\cos(x)}{1} \cdot \frac{\sin(x)}{\cos(x)}} + 2x\sin(x)\tan(x)$$

$$= x^2\sin(x)\sec^2(x) + x^2\sin(x) + 2x\sin(x)\tan(x)$$
$$= x\sin(x)[x\sec^2(x) + x + 2\tan(x)]$$

18 prove $\frac{d}{dx}(\sec(x)) = \sec(x)\tan(x)$

$$\frac{d}{dx}(\sec(x)) = \frac{d}{dx}\frac{1}{\cos(x)}$$

$$= \frac{\cos(x)\cdot 0 - 1\cdot(-\sin(x))}{\cos^2(x)}$$

$$= \frac{\sin(x)}{\cos^2(x)}$$

$$= \frac{1}{\cos(x)}\cdot\frac{\sin(x)}{\cos(x)}$$

$$= \sec(x)\tan(x)$$

20. $f(x) = \cos(x)$

$$f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\cos(x+h)-\cos(x)}{h}$$

$$= \lim_{h\to 0}\frac{\cos(x)\cos(h)-\sin(x)\sin(h)-\cos(x)}{h}$$

$$= \lim_{h\to 0}\left[\frac{\cos(x)\cos(h)-\cos(x)}{h} - \frac{\sin(x)\sin(h)}{h}\right]$$

$$= \lim_{h\to 0}\left[\cos(x)\left(\frac{\cos(h)-1}{h}\right) - \sin(x)\left(\frac{\sin(h)}{h}\right)\right]$$

$$= \lim_{h\to 0}\cos(x)\cdot\lim_{h\to 0}\frac{\cos(h)-1}{h} - \lim_{h\to 0}\sin(x)\cdot\lim_{h\to 0}\frac{\sin(h)}{h}$$

$$= \cos(x)\cdot 0 - \sin(x)\cdot 1$$

$$= -\sin(x)$$

22. $y = (1+x)\cos(x)$ point $(0,1)$

$$y' = (1+x)[-\sin(x)] + \cos(x)\cdot 1$$
$$= -\sin(x) - x\sin(x) + \cos(x)$$

at $x=0$ $\quad y' = -\sin(0) - 0\sin(0) + \cos(0)$
$$= 0 - 0 + 1$$
$$= 1$$

$$y - 1 = 1(x-0)$$
$$y = x+1$$

24. $y = \dfrac{1}{\sin(x) + \cos(x)}$ point $(0,1)$

$$y' = \frac{[\sin(x)+\cos(x)]\cdot 0 - 1[\cos(x) - \sin(x)]}{[\sin(x)+\cos(x)]^2}$$
$$= \frac{-\cos(x) + \sin(x)}{[\sin(x)+\cos(x)]^2}$$

at $x=0$ $\quad y' = \dfrac{-\cos(0) + \sin(0)}{[\sin(0) + \cos(0)]^2} = \dfrac{-1+0}{(0+1)^2} = -1$

$$y - 1 = -1(x-0)$$
$$y = -x+1$$

29. $H(\theta) = \theta \sin\theta$

$$H'(\theta) = \theta\cos(\theta) + \sin(\theta)\cdot 1 = \theta\cos(\theta) + \sin\theta$$

$$H''(\theta) = \theta[-\sin(\theta)] + \cos(\theta)\cdot 1 + \cos(\theta)$$
$$= -\theta\sin(\theta) + 2\cos(\theta)$$

34. $y = \dfrac{\cos(x)}{2+\sin(x)}$

want to find where $y'=0$

$$y' = \frac{[2+\sin(x)][-\sin(x)] - \cos(x)[\cos(x)]}{[2+\sin(x)]^2}$$
$$= \frac{-2\sin(x) - \sin^2(x) - \cos^2(x)}{[2+\sin(x)]^2}$$

Note: $\sin^2(x) + \cos^2(x) = 1$

$$= \frac{-2\sin(x) - 1}{[2+\sin(x)]^2}$$

for $y'=0$ $\quad -2\sin(x) - 1 = 0$

$$-2\sin(x) = 1$$
$$\sin(x) = \frac{-1}{2}$$
$$\left.\begin{aligned} x &= \tfrac{11\pi}{6} + 2\pi n \\ x &= \tfrac{7\pi}{6} + 2\pi n \end{aligned}\right\} \text{n is } \cancel{\text{ar}} \text{ integer}$$

$$\left.\begin{aligned} x &= \tfrac{11\pi}{6} + 2\pi n \rightarrow y = \tfrac{1}{\sqrt{3}} \\ x &= \tfrac{7\pi}{6} + 2\pi n \rightarrow y = \tfrac{-1}{\sqrt{3}} \end{aligned}\right\} \begin{aligned} &\left(\tfrac{11\pi}{6} + 2\pi n, \tfrac{1}{\sqrt{3}}\right) \\ &\left(\tfrac{7\pi}{6} + 2\pi n, \tfrac{-1}{\sqrt{3}}\right) \end{aligned}$$

36. $s = 2\cos(t) + 3\sin(t) \quad t \geq 0$

ⓐ $v(t) = s' = -2\sin(t) + 3\cos(t)$

$a(t) = v'(t) = s'' = -2\cos(t) - 3\sin(t)$

ⓑ graph $a(t)$ and $v(t)$