

6.1 The Law of Sines

- Using the Law of Sines to solve oblique triangles
- Finding two triangles with the Law of Sines
- Finding the area of a SAS triangle

Examples:

1. Solve the triangle with $a = 18.7$, $c = 16.1$, and $C = 35^\circ$.

$$A_1 = 42^\circ, B_1 = 103^\circ, b_1 = 27.4$$

$$A_2 = 138^\circ, B_2 = 7^\circ, b_2 = 3.4$$

2. Find the area of the triangle with $C = 115^\circ$, $a = 4$, and $b = 5$ yards.

$$A = 9 \text{ yd}^2$$

6.2 The Law of Cosines

- Using the Law of Cosines to solve oblique triangles
- Solving applied problems using the Law of Cosines
- Finding the area of a triangle (Heron's Formula)

Examples:

3. Solve the triangle with $a = 8$, $b = 6$, and $c = 4$.

$$A = 104^\circ, B = 47^\circ, C = 29^\circ$$

4. Two points A and B are on opposite sides of a building. A surveyor selects a third point C to place a transit. Point C is 52 feet from point A and 74 feet from point B. The angle ACB is 51° . How far apart are points A and B?

$$57.8 \text{ ft.}$$

5. Find the area of the triangle with $a = 8$, $b = 13$, and $c = 12$ meters.

$$A = 47 \text{ m}^2$$

6.3 Polar Coordinates

- Plotting points in the polar coordinate system
- Finding multiple representations for polar coordinates
- Converting between polar and rectangular coordinates
- Converting between polar and rectangular equations

Examples:

6. Plot the point $(4, \frac{\pi}{3})$ in the polar coordinate system. Then find another representation for the point where $r < 0$ and $2\pi < \theta < 4\pi$.

$$(-4, \frac{10\pi}{3})$$

7. Find the rectangular coordinates of the polar point $(9, \frac{3\pi}{4})$.

$$(-\frac{9\sqrt{2}}{2}, \frac{9\sqrt{2}}{2})$$

8. Find the polar coordinates of the point $(-2\sqrt{2}, -2\sqrt{2})$.

$$(4, 225^\circ)$$

9. Convert the rectangular equation to a polar equation that expresses r in terms of θ .

$$4x + 3y - 6 = 0$$

$$r = \frac{6}{4 \cos \theta + 3 \sin \theta}$$

6.4 Graphs of Polar Equations

- Matching graphs of various polar equations, including circles, rose curves, lemniscates, and limaçons, to their equations.

Examples:

10. Sketch the graph of $r = 3\sin(7\theta)$.

**The graph is a rose curve with 7 petals. Set calculator to POL/RADIAN mode.*

6.5 Complex Numbers in Polar Form

- Writing complex numbers in polar form
- Converting complex numbers from polar to rectangular form
- Finding products and quotients of complex numbers in polar form
- Finding powers and roots of complex numbers in polar form

Examples:

11. Write the complex number $2 - 2i$ in polar form. Express the argument in radians.

$$2\sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$$

12. Write the complex number $-5(\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3})$ in rectangular form.

$$\frac{5}{2} + \frac{-5\sqrt{3}}{2}i$$

13. Find a) the product and b) the quotient of the complex numbers. Leave the answer in polar form.

$$z_1 = \sqrt{3} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$$

$$z_2 = \sqrt{6} \left(\cos \frac{9\pi}{4} + i \sin \frac{9\pi}{4} \right)$$

a) $z_1 z_2 = 3\sqrt{2}(\cos 4\pi + i \sin 4\pi)$

b) $\frac{z_1}{z_2} = \frac{\sqrt{2}}{2} \left(\cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} \right)$

14. Use DeMoivre's Theorem to find the indicated power of the complex number. Write the answer in rectangular form.

$$\left[2\sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right) \right]^5$$

$$-128 + 128i$$

15. Find the complex cube roots of $216(\cos 321^\circ + i \sin 321^\circ)$ in polar form.

$$6(\cos 107^\circ + i \sin 107^\circ), 6(\cos 227^\circ + i \sin 227^\circ), 6(\cos 347^\circ + i \sin 347^\circ)$$

6.6 Vectors

- Find the magnitude of a vector
- Perform operations with vectors
- Write a vector in terms of \mathbf{i} and \mathbf{j}
- Find unit vectors
- Write a vector in terms of magnitude and direction

Examples:

16. Let \mathbf{v} be the vector from initial point $P_1(6, 4)$ to terminal point $P_2(-5, -4)$. Write \mathbf{v} in terms of \mathbf{i} and \mathbf{j} .

$$\mathbf{v} = -11\mathbf{i} - 8\mathbf{j}$$

17. Find $\|\mathbf{u} + \mathbf{v}\|$ if $\mathbf{u} = -7\mathbf{i} + \mathbf{j}$ and $\mathbf{v} = 8\mathbf{i} + \mathbf{j}$.

$$\sqrt{5}$$

18. Write a vector in terms of \mathbf{i} and \mathbf{j} with magnitude $\|\mathbf{v}\| = 6$ and $\theta = 240^\circ$.

$$\mathbf{v} = -3\mathbf{i} - 3\sqrt{3}\mathbf{j}$$

6.7 The Dot Product

- Find the dot product of two vectors
- Find the angle between two vectors
- Determine if two vectors are orthogonal or parallel

Examples:

19. Find $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w})$ if $\mathbf{u} = 6\mathbf{i} + 10\mathbf{j}$, $\mathbf{v} = -2\mathbf{i} - 8\mathbf{j}$, and $\mathbf{w} = -4\mathbf{i} - 2\mathbf{j}$.

$$-136$$

20. Find the angle between the vectors $\mathbf{u} = -\mathbf{i} + 4\mathbf{j}$, $\mathbf{v} = 2\mathbf{i} - 5\mathbf{j}$.

$$172.2^\circ$$