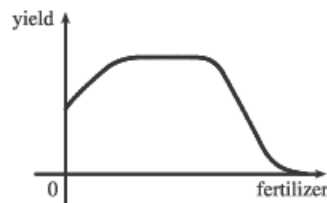

Assignment #1.4b Solutions

1. (a) When $x = 2$, $y \approx 2.7$. Thus, $f(2) \approx 2.7$. (b) $f(x) = 3 \Rightarrow x \approx 2.3, 5.6$
(c) The domain of f is $-6 \leq x \leq 6$, or $[-6, 6]$. (d) The range of f is $-4 \leq y \leq 4$, or $[-4, 4]$.
(e) f is increasing on $[-4, 4]$, that is, on $-4 \leq x \leq 4$.
(f) f is odd since its graph is symmetric about the origin.
2. (a) This curve is *not* the graph of a function of x since it *fails* the Vertical Line Test.
(b) This curve is the graph of a function of x since it *passes* the Vertical Line Test. The domain is $[-3, 3]$ and the range is $[-2, 3]$.

3. $f(x) = x^2 - 2x + 3$, so $f(a+h) = (a+h)^2 - 2(a+h) + 3 = a^2 + 2ah + h^2 - 2a - 2h + 3$, and

$$\frac{f(a+h) - f(a)}{h} = \frac{(a^2 + 2ah + h^2 - 2a - 2h + 3) - (a^2 - 2a + 3)}{h} = \frac{h(2a + h - 2)}{h} = 2a + h - 2.$$

4. There will be some yield with no fertilizer, increasing yields with increasing fertilizer use, a leveling-off of yields at some point, and disaster with too much fertilizer use.

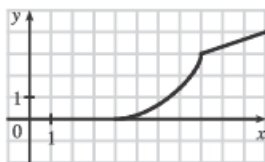


5. $f(x) = 2/(3x - 1)$. Domain: $3x - 1 \neq 0 \Rightarrow 3x \neq 1 \Rightarrow x \neq \frac{1}{3}$. $D = (-\infty, \frac{1}{3}) \cup (\frac{1}{3}, \infty)$
Range: all reals except 0 ($y = 0$ is the horizontal asymptote for f). $R = (-\infty, 0) \cup (0, \infty)$
6. $g(x) = \sqrt{16 - x^4}$. Domain: $16 - x^4 \geq 0 \Rightarrow x^4 \leq 16 \Rightarrow |x| \leq \sqrt[4]{16} \Rightarrow |x| \leq 2$. $D = [-2, 2]$
Range: $y \geq 0$ and $y \leq \sqrt{16} \Rightarrow 0 \leq y \leq 4$. $R = [0, 4]$
7. $y = 1 + \sin x$. Domain: \mathbb{R} .
Range: $-1 \leq \sin x \leq 1 \Rightarrow 0 \leq 1 + \sin x \leq 2 \Rightarrow 0 \leq y \leq 2$. $R = [0, 2]$
8. $y = F(t) = 3 + \cos 2t$. Domain: \mathbb{R} . $D = (-\infty, \infty)$
Range: $-1 \leq \cos 2t \leq 1 \Rightarrow 2 \leq 3 + \cos 2t \leq 4 \Rightarrow 2 \leq y \leq 4$. $R = [2, 4]$

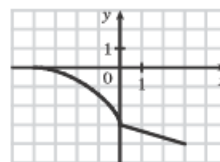
Assignment #1.4b Solutions

9. (a) To obtain the graph of $y = f(x) + 8$, we shift the graph of $y = f(x)$ up 8 units.
- (b) To obtain the graph of $y = f(x + 8)$, we shift the graph of $y = f(x)$ left 8 units.
- (c) To obtain the graph of $y = 1 + 2f(x)$, we stretch the graph of $y = f(x)$ vertically by a factor of 2, and then shift the resulting graph 1 unit upward.
- (d) To obtain the graph of $y = f(x - 2) - 2$, we shift the graph of $y = f(x)$ right 2 units (for the “-2” inside the parentheses), and then shift the resulting graph 2 units downward.
- (e) To obtain the graph of $y = -f(x)$, we reflect the graph of $y = f(x)$ about the x -axis.
- (f) To obtain the graph of $y = 3 - f(x)$, we reflect the graph of $y = f(x)$ about the x -axis, and then shift the resulting graph 3 units upward.

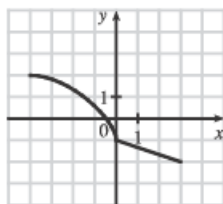
10. (a) To obtain the graph of $y = f(x - 8)$, we shift the graph of $y = f(x)$ right 8 units.



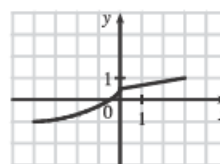
- (b) To obtain the graph of $y = -f(x)$, we reflect the graph of $y = f(x)$ about the x -axis.



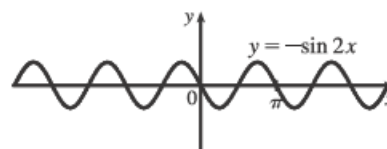
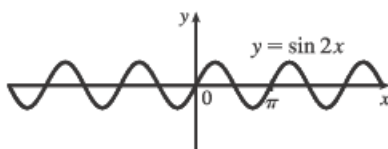
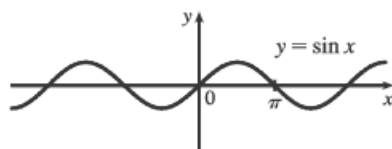
- (c) To obtain the graph of $y = 2 - f(x)$, we reflect the graph of $y = f(x)$ about the x -axis, and then shift the resulting graph 2 units upward.



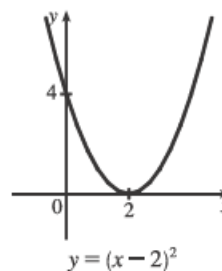
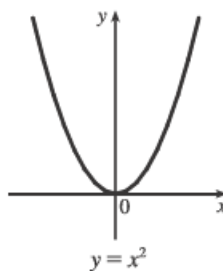
- (d) To obtain the graph of $y = \frac{1}{2}f(x) - 1$, we shrink the graph of $y = f(x)$ by a factor of 2, and then shift the resulting graph 1 unit downward.



11. $y = -\sin 2x$: Start with the graph of $y = \sin x$, compress horizontally by a factor of 2, and reflect about the x -axis.

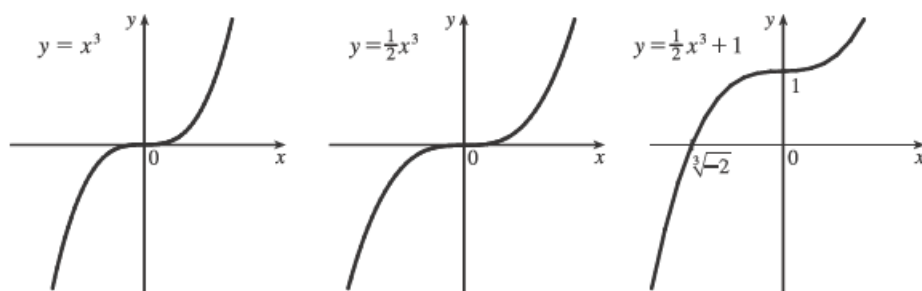


12. $y = (x - 2)^2$: Start with the graph of $y = x^2$ and shift 2 units to the right.

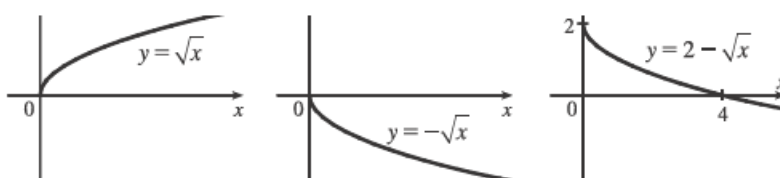


Assignment #1.4b Solutions

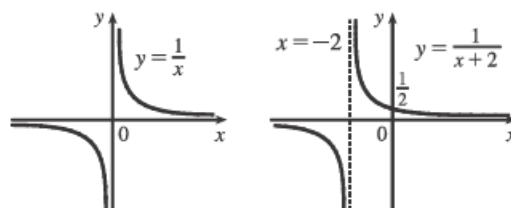
13. $y = 1 + \frac{1}{2}x^3$: Start with the graph of $y = x^3$, compress vertically by a factor of 2, and shift 1 unit upward.



14. $y = 2 - \sqrt{x}$: Start with the graph of $y = \sqrt{x}$, reflect about the x -axis, and shift 2 units upward.



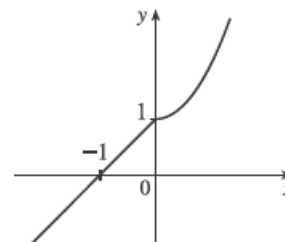
15. $f(x) = \frac{1}{x+2}$: Start with the graph of $f(x) = 1/x$ and shift 2 units to the left.



$$16. f(x) = \begin{cases} 1+x & \text{if } x < 0 \\ 1+x^2 & \text{if } x \geq 0 \end{cases}$$

On $(-\infty, 0)$, graph $y = 1 + x$ (the line with slope 1 and y -intercept 1) with open endpoint $(0, 1)$.

On $[0, \infty)$, graph $y = 1 + x^2$ (the rightmost half of the parabola $y = x^2$ shifted 1 unit upward) with closed endpoint $(0, 1)$.



17. (a) The terms of f are a mixture of odd and even powers of x , so f is neither even nor odd.
 (b) The terms of f are all odd powers of x , so f is odd.
 (c) $f(-x) = \cos((-x)^2) = \cos(x^2) = f(x)$, so f is even.
 (d) $f(-x) = 1 + \sin(-x) = 1 - \sin x$. Now $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$, so f is neither even nor odd.

18. For the line segment from $(-2, 2)$ to $(-1, 0)$, the slope is $\frac{0-2}{-1+2} = -2$, and an equation is $y - 0 = -2(x + 1)$ or, equivalently, $y = -2x - 2$. The circle has equation $x^2 + y^2 = 1$; the top half has equation $y = \sqrt{1 - x^2}$ (we have solved for positive y). Thus, $f(x) = \begin{cases} -2x - 2 & \text{if } -2 \leq x \leq -1 \\ \sqrt{1 - x^2} & \text{if } -1 < x \leq 1 \end{cases}$

Assignment #1.4b Solutions

19. $f(x) = \sqrt{x}$, $D = [0, \infty)$; $g(x) = \sin x$, $D = \mathbb{R}$.

(a) $(f \circ g)(x) = f(g(x)) = f(\sin x) = \sqrt{\sin x}$. For $\sqrt{\sin x}$ to be defined, we must have $\sin x \geq 0 \Leftrightarrow$

$x \in [0, \pi], [2\pi, 3\pi], [-2\pi, -\pi], [4\pi, 5\pi], [-4\pi, -3\pi], \dots$, so $D = \{x \mid x \in [2n\pi, \pi + 2n\pi], \text{ where } n \text{ is an integer}\}$.

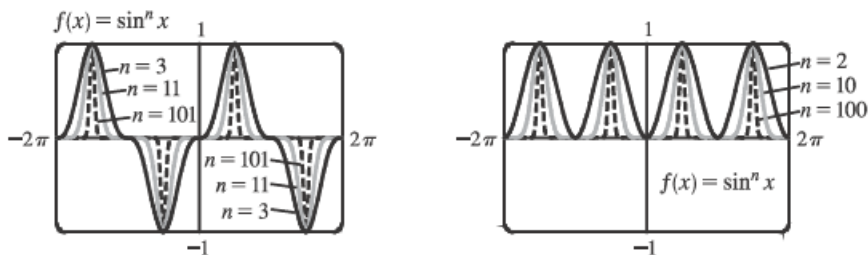
(b) $(g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = \sin \sqrt{x}$. x must be greater than or equal to 0 for \sqrt{x} to be defined, so $D = [0, \infty)$.

(c) $(f \circ f)(x) = f(f(x)) = f(\sqrt{x}) = \sqrt{\sqrt{x}} = \sqrt[4]{x}$. $D = [0, \infty)$.

(d) $(g \circ g)(x) = g(g(x)) = g(\sin x) = \sin(\sin x)$. $D = \mathbb{R}$.

20. Let $h(x) = x + \sqrt{x}$, $g(x) = \sqrt{x}$, and $f(x) = 1/x$. Then $(f \circ g \circ h)(x) = \frac{1}{\sqrt{x + \sqrt{x}}} = F(x)$.

21. The graphs of $f(x) = \sin^n x$, where n is a positive integer, all have domain \mathbb{R} . For odd n , the range is $[-1, 1]$ and for even n , the range is $[0, 1]$. For odd n , the functions are odd and symmetric with respect to the origin. For even n , the functions are even and symmetric with respect to the y -axis. As n becomes large, the graphs become less rounded and more “spiky.”

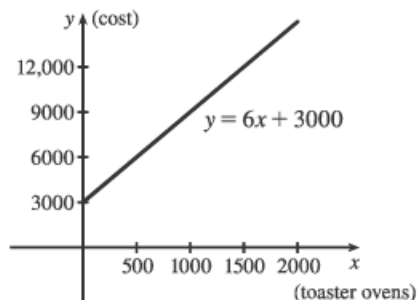


22. (a) Let x denote the number of toaster ovens produced in one week and y the associated cost. Using the points $(1000, 9000)$ and

$(1500, 12,000)$, we get an equation of a line:

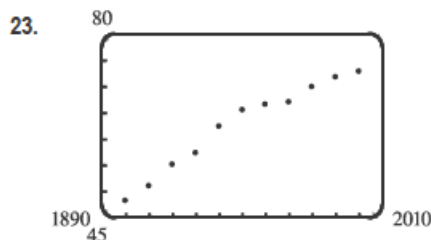
$$y - 9000 = \frac{12,000 - 9000}{1500 - 1000} (x - 1000) \Rightarrow$$

$$y = 6(x - 1000) + 9000 \Rightarrow y = 6x + 3000.$$



(b) The slope of 6 means that each additional toaster oven produced adds \$6 to the weekly production cost.

(c) The y -intercept of 3000 represents the overhead cost—the cost incurred without producing anything.



Many models appear to be plausible. Your choice depends on whether you think medical advances will keep increasing life expectancy, or if there is bound to be a natural leveling-off of life expectancy. A linear model, $y = 0.2493x - 423.4818$, gives us an estimate of 77.6 years for the year 2010.