8-8 Analyzing Decisions

Going Deeper

Essential question: *How can you use probability to help you analyze decisions?*

COMMON CORE Standards for Mathematical Content

CC.9-12.S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified . . . .*

CC.9-12.S.MD.7(+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing . . .).*

In experiments with numerical outcomes, the **expected value** (EV) is the weighted average of the numerical outcomes of a probability experiment.

The sides of a six-sided number cube are labeled 1, 1, 3, 3, 9, and 9.

A. What is the expected value of the number cube?

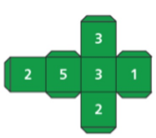
$$EV(x) = 1\left(\frac{2}{6}\right) + 3\left(\frac{2}{6}\right) + 9\left(\frac{2}{6}\right)$$

$$= \frac{2}{6} + \frac{6}{6} + \frac{18}{6}$$

$$= \frac{26}{6}$$

$$= 4\tfrac{2}{6} \text{ or } 4\tfrac{1}{3}$$

What is the expected value of rolling the six sided number cube as shown in the net below?



$$EV(x) = 1\left(\frac{1}{6}\right) + 2\left(\frac{2}{6}\right) + 3\left(\frac{2}{6}\right) + 5\left(\frac{1}{6}\right)$$

$$\frac{1}{6} + \frac{4}{6} + \frac{6}{6} + \frac{5}{6}$$

$$\frac{16}{6} = 2\tfrac{4}{6} = 2\tfrac{2}{3}$$

On a mountain, it takes Sam 2 hours to climb the southern route, unless there is ice, which increases the time to 4 hours. It takes him 2.5 hours to climb the eastern route, unless there is ice, which increases the time to 3 hours. If the chance of ice is 20% on the southern route and 40% on the eastern route, which route should Sam take if he wants to finish the climb as soon as possible?

$$EV(\text{Southern}) = .20(4) + .80(2) = 2.4 \leftarrow$$

$$EV(\text{Eastern}) = .40(3) + .60(2.5) = 2.7$$

$$\boxed{\text{Southern}}$$

Jack can take one of three routes to work each day. Route A takes 16 minutes, Route B takes 10 minutes, and Route C takes 20 minutes. There is a 40% chance he will encounter an accident in Route A, which increases travel time to 25 minutes. There is also a 20% chance he will encounter a traffic jam if he takes Route B, which increases his travel time to 40 minutes. He has a 10% chance of experiencing a delay in Route C, which increases his travel time to 32 minutes. Which route should Jack take to work each day?

$EV(\text{Route A}) = .40(25) + .60(16) = 19.6$

$EV(\text{Route B}) = .20(40) + .80(10) = 16$

$EV(\text{Route C}) = .10(32) + .90(20) = 21.2$

$\boxed{\text{Route B}}$

Mikayla is applying to 3 colleges. She makes estimates of her chances of being accepted, and estimates of her chances of receiving financial aid from each, presented below:

At which college is she most likely to be both accepted and receive financial aid?

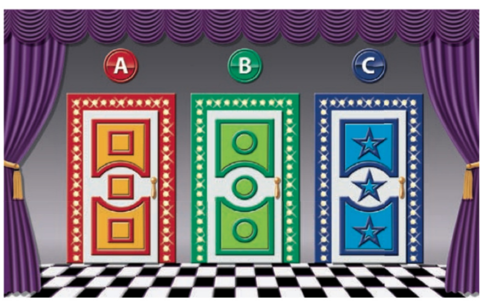
	% chance of acceptance	% chance of financial aid
College A	75%	30%
College B	65%	40%
College C	70%	45%

$EV(\text{College A}) = .75(.30) = .225$

$EV(\text{College B}) = .65(.40) = .26$

$EV(\text{College C}) = .70(.45) = .315$

$\boxed{\text{College C}}$



Two of the doors hide goats, while the third door hides a prize. Once the contestant picks a door, the host, who knows the contents behind each door, opens one of the other doors to reveal a goat. If both remaining doors contain a goat, the host chooses randomly which door to reveal. The host then offers the contestant a chance to stay with their original choice of door or switch his choice to the other remaining door. What should the contestant do?

Assume the contestant picks door A initially.

Contents Behind Each Door			Results	
Door A	Door B	Door C	Result if door is switched	Result if door is not switched
Car	Goat	Goat	Goat	Car
Goat	Car	Goat	Car	Goat
Goat	Goat	Car	Car	Goat

Notice that if the contestant switches doors, they have a $\frac{2}{3}$ chance of winning the prize while they only have a $\frac{1}{3}$ chance of winning the car if they keep the door they initially chose. The expected value of switching is twice that of staying with the original choice!

If you were in the game show, would you switch doors?

In a TV game show, a car key is hidden in one of five bags. The other bags contain fake keys. Once the contestant picks a bag, ~~the host~~, knowing where the key is located, opens a bag with a fake key. As the contestant answers questions correctly, he continues to open bags with fake keys until two bags remain: one with the car key and one with a fake key. At this time, he offers the contestant a chance to switch bags. Find the expected value of sticking with the original bag and the expected value of switching bags.

A	B	C	D	E	S	NS
Y	N	N	N	N	N	Y
N	Y	N	N	N	Y	N
N	N	Y	N	N	Y	N
N	N	N	Y	N	Y	N
N	N	N	N	Y	Y	N

$EV(\text{sticking}) = \frac{1}{5}$

$EV(\text{switching}) = \frac{4}{5}$