Name: ____________________ Date: __________ Row: ____ Period: ____

DIFFERENTIATION

Work through the FRQ on a separate sheet of paper and show all your work for the MC questions.

MULTIPLE-CHOICE QUESTIONS

Calculators may not be used for the following questions.

1. What does the limit statement $\lim_{x \to 1} \frac{\ln(x+1) - \ln 2}{x-1}$ represent?

   (A) 0
   (B) $\frac{d}{dx}[\ln(x+1)]$
   (C) $f'(1)$, if $f(x) = \ln(x+1)$
   (D) 1
   (E) The limit does not exist.

2. Find the derivative of the function $y = \frac{4}{x^3}$.

   (A) $-4x^2$
   (B) $-\frac{12}{x^2}$
   (C) $\frac{12}{x^2}$
   (D) $\frac{12}{x^4}$
   (E) $-\frac{12}{x^4}$

3. Find $\frac{dy}{dx}$ if $3xy = 4x + y^2$.

   (A) $\frac{4-3y}{2y-3x}$
   (B) $\frac{3x-4}{2x}$
   (C) $\frac{3y-x}{2}$
   (D) $\frac{3y-4}{2y-3x}$
   (E) $\frac{4+3y}{2y+3x}$

4. Find $\frac{dy}{dx}$ for $e^{x+y} = y$.

   (A) $\frac{e^{x+y}}{\left(1-e^{x+y}\right)}$
   (B) $\frac{e^{x+y}}{\left(1+e^{x+y}\right)}$
   (C) $\frac{e^{x+y}}{\left(e^{x+y}-1\right)}$
   (D) e^{x+y}
   (E) $2e^{x+y}$

5. If the nth derivative of y is denoted as $y^{(n)}$ and $y = -\sin x$, then $y^{(7)}$ is the same as

(A) y

(B) $\frac{dy}{dx}$

(C) $\frac{d^2y}{dx^2}$

(D) $\frac{d^3y}{dx^3}$

(E) None of these

6. Find the second derivative of $f(x)$ if $f(x) = (2x+3)^4$.

(A) $4(2x+3)^3$

(B) $8(2x+3)^3$

(C) $12(2x+3)^2$

(D) $24(2x+3)^2$

(E) $48(2x+3)^2$

7. Find $\frac{dy}{dx}$ for $y = 4\sin^2(3x)$.

(A) $8\sin(3x)$

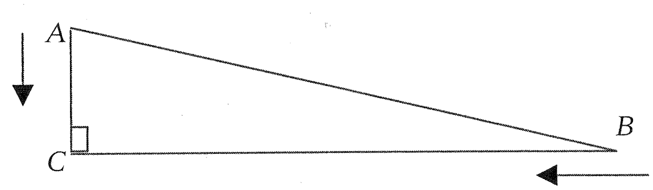
(B) $24\sin(3x)$

(C) $8\sin(3x)\cos(3x)$

(D) $12\sin(3x)\cos(3x)$

(E) $24\sin(3x)\cos(3x)$

8. In right triangle ΔABC, point A is moving along a leg of the right triangle toward point C at a rate of $\frac{1}{2}$ cm/sec and point B is moving toward point C at a rate of $\frac{1}{3}$ cm/sec along a line containing the other leg of the right triangle, as illustrated in the triangle shown below. What is the rate of change in the area of ΔABC, with respect to time, at the instant when $AC = 15$ cm and $BC = 20$ cm?



(A) -0.0833 cm^2/sec

(B) -0.4167 cm^2/sec

(C) -0.8333 cm^2/sec

(D) -7.5 cm^2/sec

(E) -15 cm^2/sec

9. If $\ln y = (\ln x)^2 + 2$, find $\dfrac{dy}{dx}$ in terms of x and y.

(A) $y\left[2\ln(x)+\dfrac{1}{x}\right]$

(B) $y\left[\left(\dfrac{2}{x}\right)\ln(x)\right]$

(C) $\left(\dfrac{2}{x}\right)\ln(x)$

(D) $\dfrac{2(\ln x)}{x}+2$

(E) $y\left[\dfrac{2(\ln x)}{x}+2\right]$

10. If $f(2) = -3$, $f'(2) = \dfrac{3}{4}$, and $g(x) = f^{-1}(x)$, what is the equation of the tangent line to $g(x)$ at $x = -3$?

(A) $y-2=\dfrac{-3}{4}(x+3)$

(B) $y+2=\dfrac{-3}{4}(x-3)$

(C) $y-2=\dfrac{-4}{3}(x+3)$

(D) $y+2=\dfrac{4}{3}(x-3)$

(E) $y-2=\dfrac{4}{3}(x+3)$

11. For what positive value of x does the tangent line to the curve $y = \ln(1 - x)$ intersect the y-axis at the point (0, 2)?

(A) 0.382
(B) 0.547
(C) 0.667
(D) 0.722
(E) 0.778

A calculator may not be used for the following questions.

12. For what values of a and c is the piecewise function

$f(x)=\begin{cases} ax^2+\sin x, & x \le \pi \\ 2x-c, & x > \pi \end{cases}$ differentiable?

(A) $a=\dfrac{3\pi}{2}$ and $c=\dfrac{\pi}{2}$

(B) $a=\dfrac{3}{2\pi}$ and $c=\dfrac{7\pi}{2}$

(C) $a=\dfrac{3}{2\pi}$ and $c=-\dfrac{\pi}{2}$

(D) $a=\dfrac{3}{2\pi}$ and $c=\dfrac{\pi}{2}$

(E) $a=\dfrac{3\pi}{2}$ and $c=\dfrac{2}{\pi}$

13. If $y = \tan^{-1}(x^2 + 3x)$, then $\frac{dy}{dx} =$

(A) $\dfrac{1}{1+(x^2+3x)^2}$

(B) $\dfrac{1}{x^2+3x+1}$

(C) $\dfrac{2x+3}{1+(x^2+3x)^2}$

(D) $\dfrac{2x+3}{(x^2+3x)^2}$

(E) $\dfrac{x^2+3x}{1+(x^2+3x)^2}$

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	1	–2	4
2	5	3	1	–4
3	2	1	–2	1
4	4	–3	2	–1

14. Selected function and derivative values for the differentiable functions $f(x)$ and $g(x)$ are given in the table above. If $p(x) = x \cdot f(x) - g(3x-2)$, then $p'(2) =$

(A) 11
(B) 10
(C) 8
(D) 6
(E) 4

15. When the height of a cylinder is 12 cm and the radius is 4 cm, the circumference of the cylinder is increasing at a rate of $\frac{\pi}{4}$ cm/min, and the height of the cylinder is increasing four times faster than the radius. How fast is the volume of the cylinder changing?

(A) $\frac{\pi}{2}$ cm^3/min

(B) 4π cm^3/min

(C) 12π cm^3/min

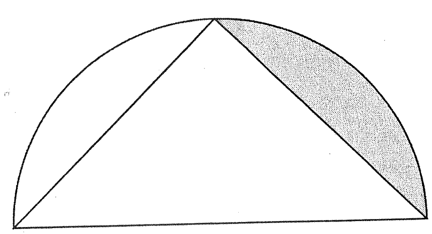
(D) 20π cm^3/min

(E) 80π cm^3/min

FREE-RESPONSE QUESTION

A calculator may be used for this question.

1. An isosceles triangle is inscribed in a semicircle, as shown in the diagram, and it continues to be inscribed as the semicircle changes size. The area of the semicircle is increasing at the rate of 1 cm²/sec when the radius of the semicircle is 3 cm.



a. How fast is the radius of the semicircle increasing when the radius is 3 cm? Include units in your answer.

b. How fast is the perimeter of the semicircle increasing when the radius is 3 cm? Include units in your answer.

c. How fast is the area of the isosceles triangle increasing when the radius is 3 cm? Include units in your answer.

d. How fast is the shaded region increasing when the radius is 3 cm? Include units in your answer.