14.3 Day 1 Notes: Matrices & Linear Systems

Properties Summary for Matrices

	Addition	Multiplication
Commutative Property	A+B = B+A	AB $\neq$ BA
Associative Property	A+(B+C)=(A+B)+C	A(BC) = (AB)C
Distributive	A(B+C) = AB + AC	

$A = \begin{bmatrix} 1 & -3 \\ 5 & 4 \end{bmatrix}$

$B = \begin{bmatrix} -2 & -7 \\ 6 & 9 \end{bmatrix}$

$C = \begin{bmatrix} 4 & -1 \\ 2 & 3 \end{bmatrix}$

Working with 2x2 "square" matrices, what is the **identity** for addition? Multiplication?

$A + \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} = A$

$A\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} = A$

$B + \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} = B$

$B\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} = B$

$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$

identity for addition

$\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} = \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix}$

identity for multiplication

Working with 2x2 "square" matrices, what is the **inverse** for addition? Multiplication?

$A + \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ Additive inverse: think A + (-A) = 0

ex: $\begin{bmatrix} 1 & -3 \\ 5 & 4 \end{bmatrix} + \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$A\begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ Multaplicative inverse: think $(A)(A^{-1}) = I_{2x2}$

Def: If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, where | A | = ad - bc

↖ | A | denotes "the determinant of A"

ex 1: Find inverses of matrices B = $\begin{bmatrix} -2 & -7 \\ 6 & 9 \end{bmatrix}$, C = $\begin{bmatrix} 1 & 3 \\ -2 & -7 \end{bmatrix}$, & D = $\begin{bmatrix} 8 & 6 \\ 4 & 3 \end{bmatrix}$

B^{-1} =

C^{-1} =

D^{-1} =

Isolating x in a real number equation versus a matrix equation:

Given	$5x = 15$	$AX = B$	$XA = B$
Mult. by inverse	$(1/5)5x = (1/5)15$	$A^{-1}AX = A^{-1}B$	$XAA^{-1} = BA^{-1}$
Inverse Prop.	$(1)x = 15/5$	$IX = A^{-1}B$	$XI = BA^{-1}$
Identity Prop.	$x = 3$	$X = A^{-1}B$	$X = BA^{-1}$

The number **1** denotes the multaplicative real number identity ↖ ↗ Matrix **I** denotes the Multaplicative Identity Matrix

ex 2: Solve the matrix equation

$AX - B = C$

$$\begin{bmatrix} 7 & -2 \\ -4 & 1 \end{bmatrix} X - \begin{bmatrix} 3 & -5 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 2 & -3 \end{bmatrix}$$

$AX = C + B$

$A^{-1}AX = A^{-1}(C + B)$

$IX = A^{-1}(C + B)$

$X = A^{-1}(C + B)$

14.3 Day 2 Notes: Matrices & Linear Systems

Note: Use T.I. for HW page 526 # 21

ex 1: Given matrix $A = \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$, find $A^{-1} =$

ex 2: Solve $\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} X + \begin{bmatrix} 7 & -1 \\ 2 & -3 \end{bmatrix} = \begin{bmatrix} -1 & 4 \\ 3 & -2 \end{bmatrix}$

ex 3: Use matrices to solve the linear system of equations $5x + 2y = -3$
$4x + y = 6$

Write as a matrix equation: A X = B

$$\begin{bmatrix} 5 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 6 \end{bmatrix}$$

Need the inverse: $A^{-1} =$

ex 4: Use matrices to solve the linear system of equations 4x + 2y = -11
7x - 3y = 8

Matrix equation: $\begin{bmatrix} 4 & 2 \\ 7 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -11 \\ 8 \end{bmatrix}$

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Auto dealership profit computation

Matrix S is 3 x 2 & shows cars sold in March by each car dealer

Matrix P is 1 x 3 & shows profit for each type of car

SP is not defined b/c you cannot multiply a 3x2 by a 1x3 since the inside dimensions do not match

PS is defined b/c you can multiply a 1x3 by a 3x2 since the inside dimensions match

PS = [18(400)+24(650)+16(900)   15(400)+17(650)+20(900)]

= **[37,200   35,050]** and this represents the number of cars sold by each dealer in March times the profit for each type of car which equals **each dealer's profit for March**

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Business application

a. Matrix P = Production schedule for 3 calculator models 3x3
M = # of components needed for ea. model 3x3
R = # of relays needed for ea. component 2x3

PM is defined b/c the dimensions work for multiplication but if multiply the units, they aren't meaningful

b. M^tP = [[2 2 2] [1 1 5] [2 4 6]] [[500 600 600] [200 200 200] [100 300 400]] =

	Jan	Feb	Mar
A	1600	2200	2400
B	1200	2300	2800
C	2400	3800	4400

c. MR is not defined b/c the inside dimensions do not match so we cannot multiply in this order

d. RM^t = number of relays of each type needed for ea. calculator type

e. RM^tP = number of relays of each type needed per month for production

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Industrial Design competition

a. Write an equation for precision, finish, and artistry for Stephanie:
.30P + .20F + .50A = .30(8) + .20(7) + .50(9) = 2.4 + 1.4 + 4.5 = 8.3

b.

	Precision	Finish	Artistry
Steph	8	7	9
Joyce	8	6	10
Eduardo	6	8	10
Frank	9	10	7
Asabi	10	10	6
Dara	8	7	8

Weight
.30
.20
.50

6 by 3 matrix **3 by 1 matrix**

Highest weighted score: Joyce
2nd highest: Eduardo
3rd highest: Stephanie 8.3