2.7 Piecewise Functions

GOAL 1 REPRESENTING PIECEWISE FUNCTIONS

What you should learn

GOAL 1 Represent piecewise functions.

GOAL 2 Use piecewise functions to model **real-life** quantities, such as the amount you earn at a summer job in **Example 6**.

Why you should learn it

▼ To solve **real-life** problems, such as determining the cost of ordering silk-screen T-shirts in **Exs. 54 and 55**.

Up to now in this chapter a function has been represented by a single equation. In many real-life problems, however, functions are represented by a combination of equations, each corresponding to a part of the domain. Such functions are called **piecewise functions**. For example, the piecewise function given by

$$f(x) = \begin{cases} 2x - 1, & \text{if } x \le 1 \\ 3x + 1, & \text{if } x > 1 \end{cases}$$

is defined by two equations. One equation gives the values of $f(x)$ when x is less than or equal to 1, and the other equation gives the values of $f(x)$ when x is greater than 1.

EXAMPLE 1 *Evaluating a Piecewise Function*

Evaluate $f(x)$ when (**a**) $x = 0$, (**b**) $x = 2$, and (**c**) $x = 4$.

$$f(x) = \begin{cases} x + 2, & \text{if } x < 2 \\ 2x + 1, & \text{if } x \ge 2 \end{cases}$$

SOLUTION

a. $f(x) = x + 2$ — **Because 0 < 2, use first equation.**

$f(0) = 0 + 2 = 2$ — **Substitute 0 for *x*.**

b. $f(x) = 2x + 1$ — **Because 2 ≥ 2, use second equation.**

$f(2) = 2(2) + 1 = 5$ — **Substitute 2 for *x*.**

c. $f(x) = 2x + 1$ — **Because 4 ≥ 2, use second equation.**

$f(4) = 2(4) + 1 = 9$ — **Substitute 4 for *x*.**

EXAMPLE 2 *Graphing a Piecewise Function*

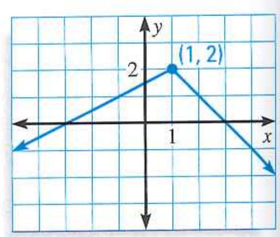
Graph this function: $f(x) = \begin{cases} \frac{1}{2}x + \frac{3}{2}, & \text{if } x < 1 \\ -x + 3, & \text{if } x \ge 1 \end{cases}$

SOLUTION

To the left of $x = 1$, the graph is given by $y = \frac{1}{2}x + \frac{3}{2}$.

To the right of and including $x = 1$, the graph is given by $y = -x + 3$.

The graph is composed of two rays with common initial point (1, 2).

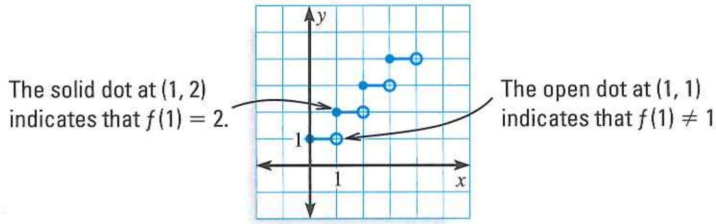


EXAMPLE 3 Graphing a Step Function

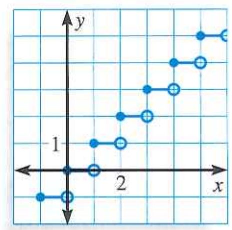
Graph this function: $f(x) = \begin{cases} 1, & \text{if } 0 \le x < 1 \\ 2, & \text{if } 1 \le x < 2 \\ 3, & \text{if } 2 \le x < 3 \\ 4, & \text{if } 3 \le x < 4 \end{cases}$

SOLUTION

The graph of the function is composed of four line segments. For instance, the first line segment is given by the equation $y = 1$ and represents the graph when x is greater than or equal to 0 and less than 1.

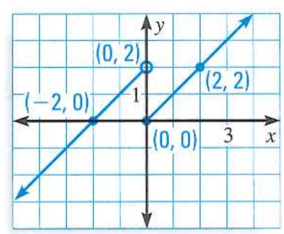


The function in Example 3 is called a **step function** because its graph resembles a set of stair steps. Another example of a step function is the *greatest integer function.* This function is denoted by $g(x) = [\![x]\!]$. For every real number x, $g(x)$ is the greatest integer less than or equal to x. The graph of $g(x)$ is shown at the right. Note that in Example 3 the function f could have been written as $f(x) = [\![x]\!] + 1, 0 \le x < 4$.



EXAMPLE 4 Writing a Piecewise Function

Write equations for the piecewise function whose graph is shown.



SOLUTION

To the left of $x = 0$, the graph is part of the line passing through $(-2, 0)$ and $(0, 2)$. An equation of this line is given by:

$$y = x + 2$$

To the right of and including $x = 0$, the graph is part of the line passing through $(0, 0)$ and $(2, 2)$. An equation of this line is given by:

$$y = x$$

▶ The equations for the piecewise function are:

$$f(x) = \begin{cases} x + 2, & \text{if } x < 0 \\ x, & \text{if } x \ge 0 \end{cases}$$

Note that $f(x) = x + 2$ does *not* correspond to $x = 0$ because there is an *open* dot at $(0, 2)$, but $f(x) = x$ *does* correspond to $x = 0$ because there is a *solid* dot at $(0, 0)$.

GOAL 2 USING PIECEWISE FUNCTIONS IN REAL LIFE

REAL LIFE
Urban Parking

EXAMPLE 5 *Using a Step Function*

a. Write and graph a piecewise function for the parking charges shown on the sign.

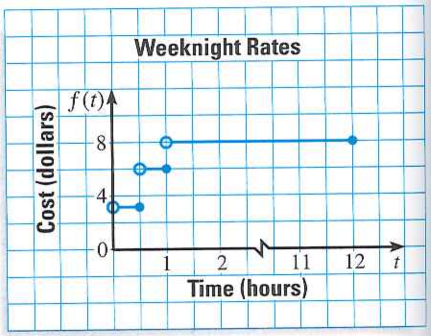
b. What are the domain and range of the function?

Garage Rates (Weekends)
$3 per half hour
$8 maximum for 12 hours

SOLUTION

a. For times up to one half hour, the charge is \$3. For each additional half hour (or portion of a half hour), the charge is an additional \$3 until you reach \$8. Let t represent the number of hours you park. The piecewise function and graph are:

$$f(t) = \begin{cases} 3, & \text{if } 0 < t \leq 0.5 \\ 6, & \text{if } 0.5 < t \leq 1 \\ 8, & \text{if } 1 < t \leq 12 \end{cases}$$



b. The domain is $0 < t \leq 12$, and the range consists of 3, 6, 8

REAL LIFE
Wages

EXAMPLE 6 *Using a Piecewise Function*

You have a summer job that pays time and a half for overtime. That is, if you work more than 40 hours per week, your hourly wage for the extra hours is 1.5 times your normal hourly wage of \$7.

a. Write and graph a piecewise function that gives your weekly pay P in terms of the number h of hours you work.

b. How much will you get paid if you work 45 hours?

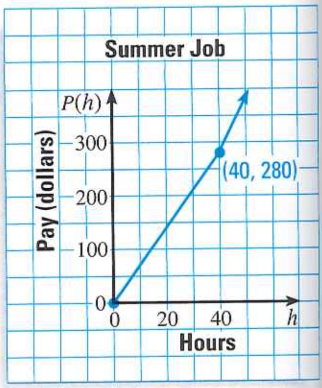
SOLUTION

a. For up to 40 hours your pay is given by $7h$.
For over 40 hours your pay is given by:

$$7(40) + 1.5(7)(h - 40) = 10.5h - 140$$

▶ The piecewise function is:

$$P(h) = \begin{cases} 7h, & \text{if } 0 \leq h \leq 40 \\ 10.5h - 140, & \text{if } h > 40 \end{cases}$$



The graph of the function is shown. Note that for up to 40 hours the rate of change is \$7 per hour, but for over 40 hours the rate of change is \$10.50 per hour.

b. To find how much you will get paid for working 45 hours, use the equation $P(h) = 10.5h - 140$.

$$P(45) = 10.5(45) - 140 = 332.5$$

▶ You will earn \$332.50.